Chapter 1 to 3 test

45 marks

1. Find the set of values of *k* for which the line $y = k(4x - 3)$ does not intersect the curve $y = 4x^2 + 8x - 8$.

[5]

$b^2 - 4ac < 0$

$k(4x-3) = 4x^2 + 8x - 8$

$4kx - 3k = 4x^2 + 8x - 8$

$4x^2 + 8x - 8 - 4kx + 3k = 0$

$a = 4, \quad b = 8 - 4k, \quad c = 3k - 8$

$b^2 - 4ac < 0$

$(8-4k)^2 - 4(4)(3k-8) < 0$

$64 - 64k + 16k^2 - 48k + 128 < 0$

$16k^2 - 112k + 192 < 0$

$3 < k < 4$

2. The functions f and g are defined by

$f(x) = \frac{2x}{x+1}$ for $x > 0$

$g(x) = \sqrt{x+1}$ for $x > -1$ $, y > 0$

a. Find $fg(8)$.

$f(3) = \frac{6}{4} = \frac{3}{2}$

[2]

b. Find an expression for $f^2(x)$, giving your answer in the form $\frac{ax}{bx+c}$ where a,b and c are integers to be found.

$ff(x) = f\left(\frac{2x}{x+1}\right)$

[3]

$= \frac{2\left(\frac{2x}{x+1}\right)}{\frac{2x}{x+1}+1} = \frac{4x}{x+1} \div \frac{2x+x+1}{x+1}$

$= \frac{4x}{x+1} \div \frac{3x+1}{x+1}$

$= \frac{4x}{3x+1}$

c. Find an expression for $g^{-1}(x)$, stating its domain and range.

$y = \sqrt{x+1}$

[4]

$x = \sqrt{y+1}$

$x^2 - 1 = y$

$g^{-1}(x) = x^2 - 1, x > 0, y > -1$

d. On the same axes, sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$, indicating the geometrical relationship between the graphs.

[3]

$g^{-1}(x) = x^2 - 1$

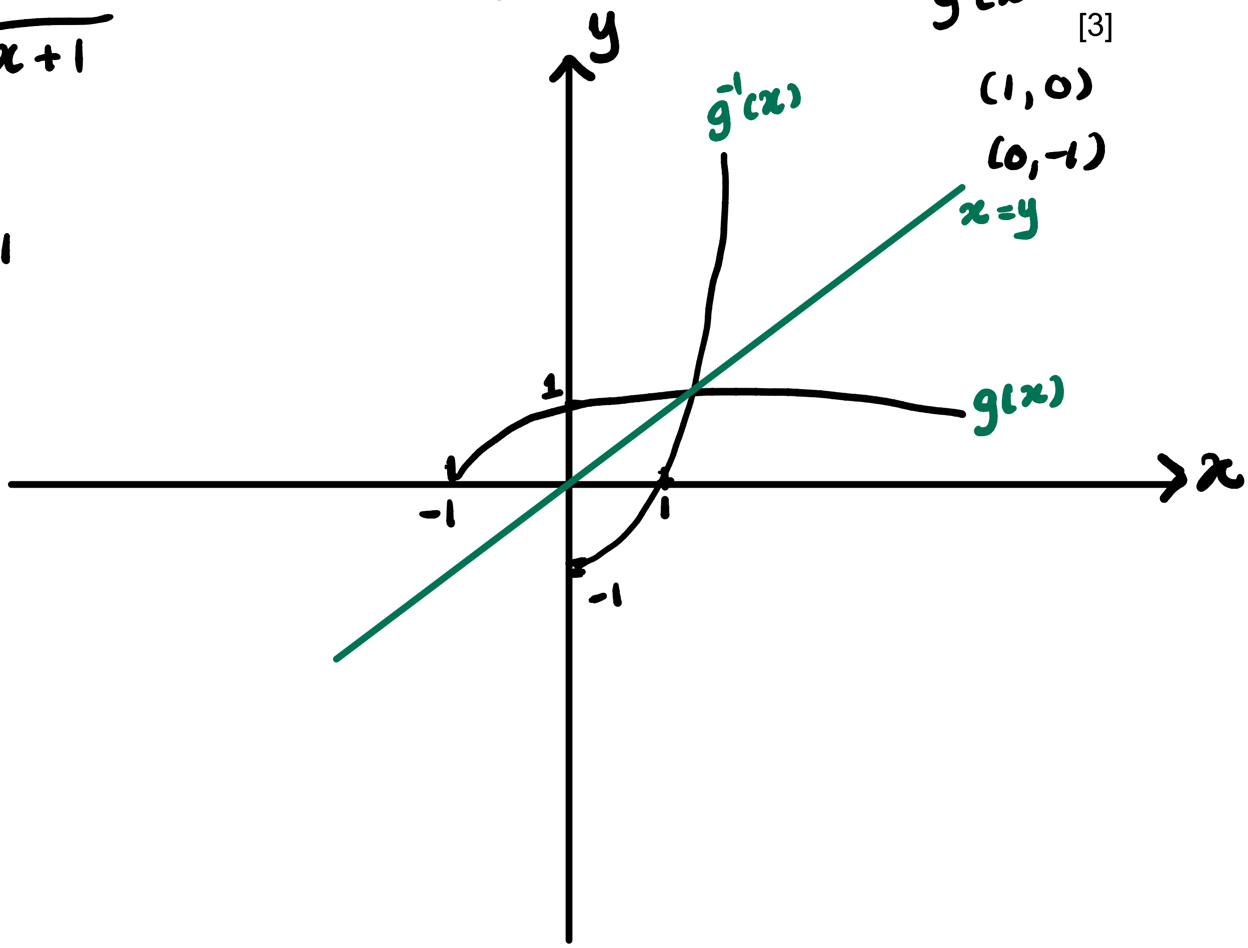
$(1, 0)$

$(0, -1)$

$y = \sqrt{x+1}$

$x = 0,\ y = 1$

$y = 0,\ x = -1$

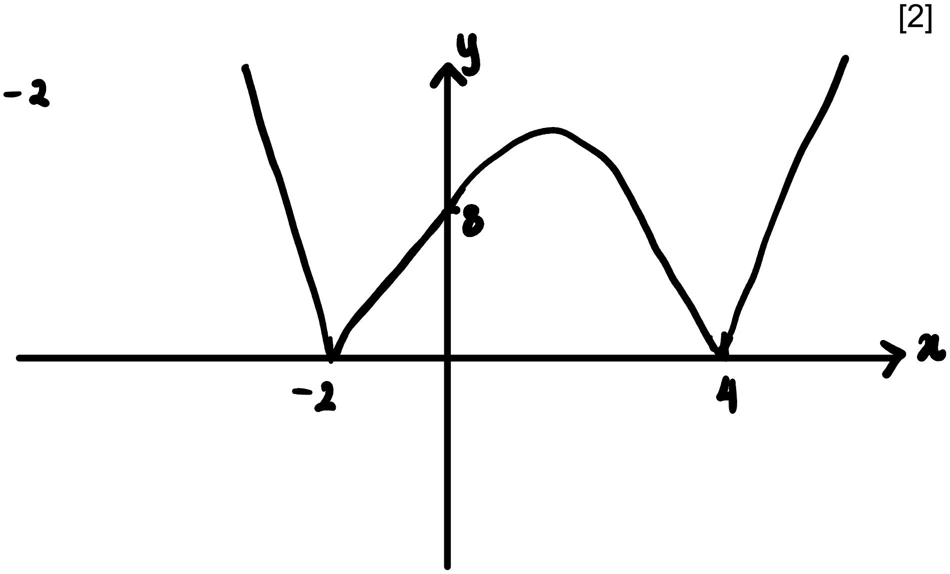


3.

a. Sketch the graph of $y = |(x - 4)(x + 2)|$ showing the coordinates of the points where the curve meets the *x*-axis.

[2]

$x = 0, y = 8$
$y = 0, x = 4$ or -2



b. Find the set of values of *k* for which $k = |(x - 4)(x + 2)|$ has four solutions.

[3]

Stationary pt : $x = \frac{4-2}{2} = \frac{2}{2} = 1$

$y = |-3 \times 3|$
$= 9$

$0 < k < 9$

4.

a. Express $2x^2 - x + 6$ in the form $p(x - q)^2 + r$ where *p*, *q* and *r* are constants to be found.

[3]

$$2x^2 - x + 6 = p(x-q)^2 + r$$
$$= p(x^2 - 2qx + q^2) + r$$
$$= px^2 - 2pqx + pq^2 + r$$

$p = 2$
$2pq = 1$
$4q = 1$
$q = \frac{1}{4}$

$pq^2 + r = 6$
$\cancel{2}^{1} \times \frac{1}{\cancel{16}_{8}} + r = 6$
$r = 6 - \frac{1}{8}$
$= \frac{47}{8}$

$$\therefore 2x^2 - x + 6 = 2\left(x - \frac{1}{4}\right)^2 + \frac{47}{8}$$

b. Hence state the least value of $2x^2 - x + 6$ and the value of *x* at which this occurs.

[2]

least value $= \frac{47}{8}$

$x = \frac{1}{4}$

5. **Do not use a calculator in this question**

a. Show that $(2\sqrt{2}+4)^2 - 8(2\sqrt{2}+3) = 0$. [2]

$L.H.S = 4\times 2 + 16\cancel{\sqrt{2}} + 16 - 16\cancel{\sqrt{2}} - 24$

$= 8 + 16 - 24$

$= 0$

$= R.H.S$

b. Solve the equation $(2\sqrt{2}+3)x^2 - (2\sqrt{2}+4)x + 2 = 0$, giving your answer in the form $a + b\sqrt{2}$ where *a* and *b* are integers. [3]

a

x

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{2\sqrt{2}+4 \pm \sqrt{(2\sqrt{2}+4)^2 - 4(2\sqrt{2}+3)(2)}}{2(2\sqrt{2}+3)}$$

$$= \frac{2\sqrt{2}+4 \pm \sqrt{(2\sqrt{2}+4)^2 - 8(2\sqrt{2}+3)}}{2(2\sqrt{2}+3)}$$

$$= \frac{2\sqrt{2}+4}{2(2\sqrt{2}+3)} = \frac{\sqrt{2}+2}{2\sqrt{2}+3} \times \frac{(2\sqrt{2}-3)}{(2\sqrt{2}-3)}$$

$$= \frac{4 - 3\sqrt{2} + 4\sqrt{2} - 6}{8-9}$$

$$= \frac{\sqrt{2}-2}{-1} = 2 - \sqrt{2}$$

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6. Find the coordinate of the point of intersection of the curve $\frac{8}{x} - \frac{10}{y} = 1$ and the line $x + y = 9$.

[6]

$8y - 10x = xy \rightarrow 72 - 8x - 10x = x(9 - x)$

$y = 9 - x$

$72 - 18x = 9x - x^2$

$x^2 - 27x + 72 = 0$

$(x - 3)(x - 24) = 0$

$x = 3$ or $x = 24$

$y = 6$ $\quad$ $y = -15$

$(3, 6)$ $\quad$ $(24, -15)$

7. Given that $2^{4x} \times 4^{y} \times 8^{x-y} = 1$ and $3^{x+y} = \frac{1}{3}$, find the value of *x* and of *y*.

[4]

$$2^{4x} \times 2^{2y} \times 2^{3x-3y} = 2^{0}$$

$$2^{7x-y} = 2^{0}$$

$$7x - y = 0 \quad —(1)$$

$$3^{x+y} = 3^{-1}$$

$$x + \cancel{y} = -1 \quad —(2)$$

$$7x - \cancel{y} = 0$$

$$8x = -1$$

$$x = -\frac{1}{8}$$

$$-\frac{1}{8} + y = -1$$

$$y = -\frac{7}{8}$$

8. Solve the inequality $9x^{2} + 2x - 1 < (x + 1)^{2}$.

[3]

$$9x^{2} + 2x - 1 < x^{2} + 2x + 1$$

$$8x^{2} - 2 < 0$$

$$8x^{2} < 2$$

$$x^{2} < \frac{1}{4}$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

Miley 22%

HML 47%

Vix 93%

William 40%

Sophie 40%

Lynn 93%

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